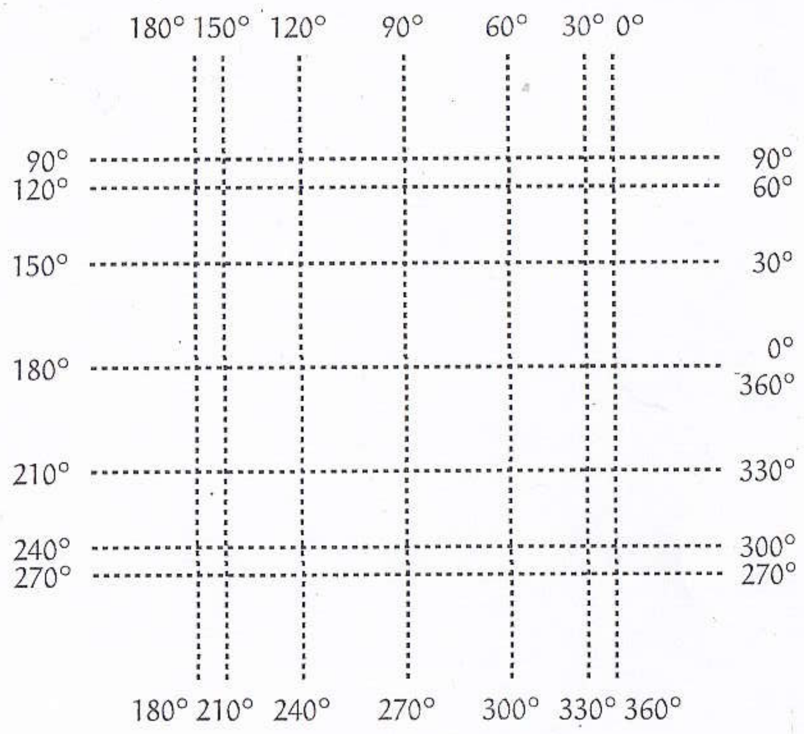
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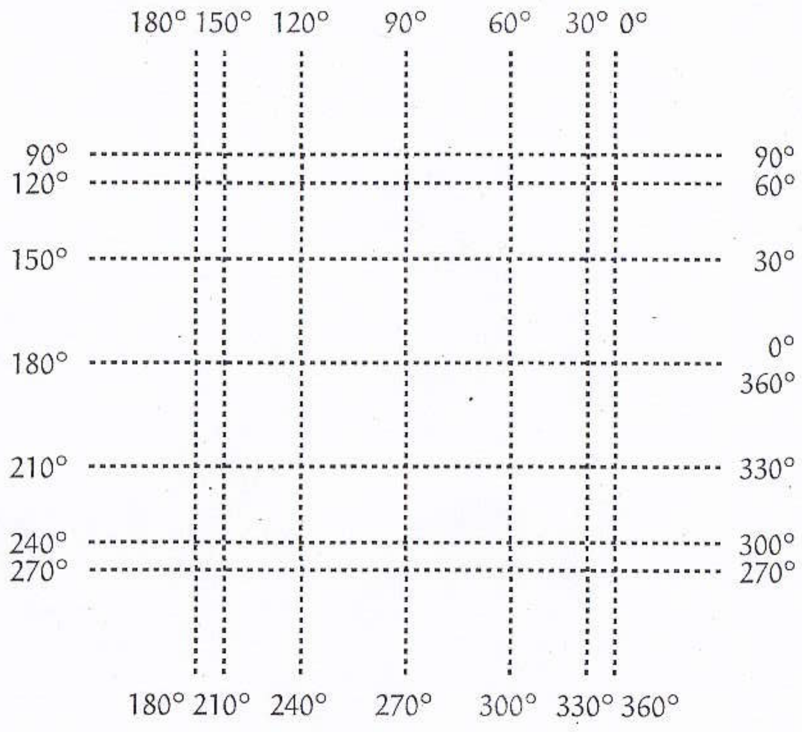
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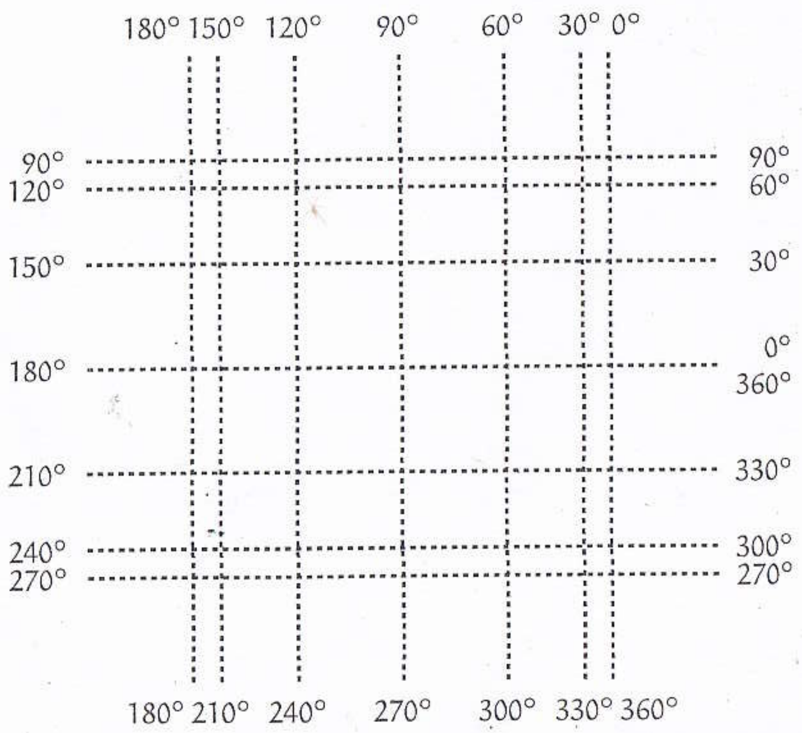
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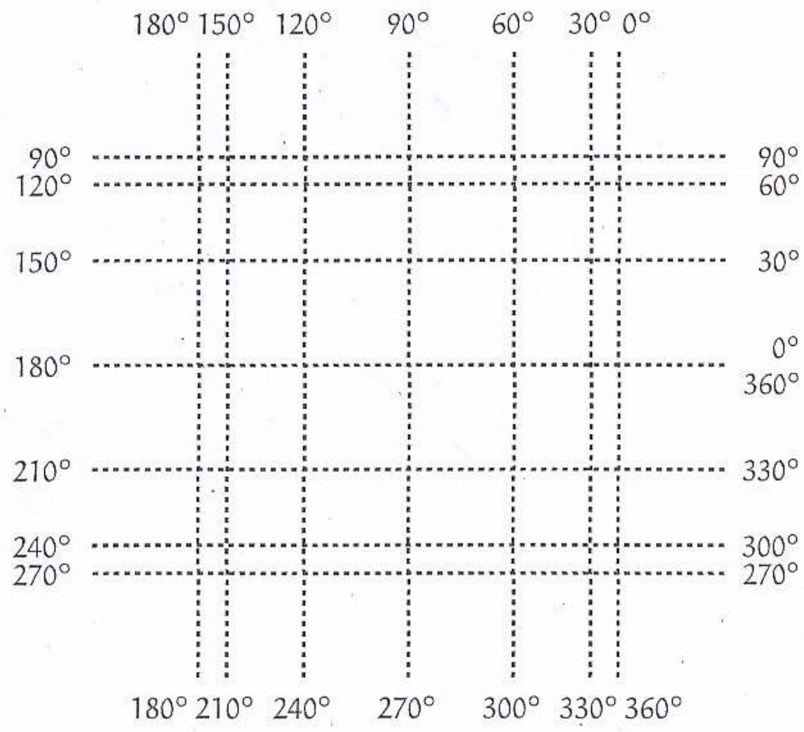
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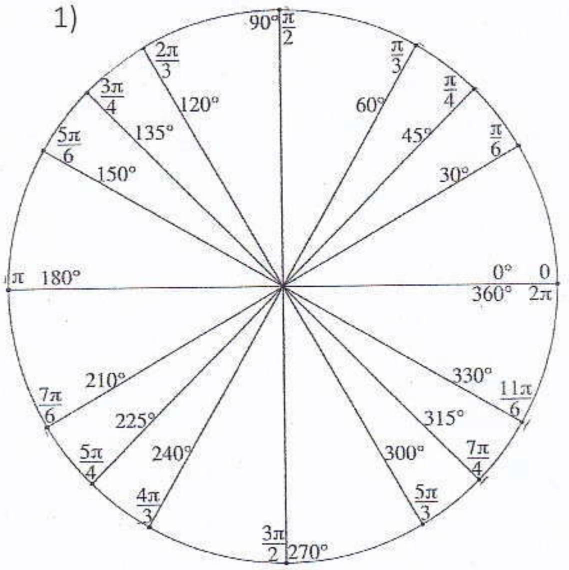
CIRCULAR FUN: ☆ STARING THE UNIT CIRCLE

Name________________________

Review of the unit circle and preparation for the activity ahead:

1)



We will create a grid system of lines using all our angles. Follow along as I do this.

You should have 7 horizontal and 7 vertical lines.

At the end of each line we will write the degree of that line. The degrees should rotate around to 360°.

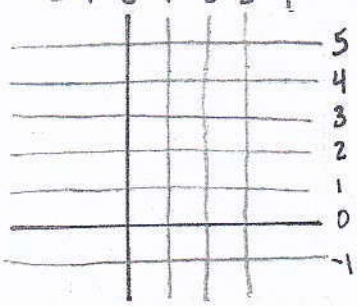
There should be 49 intersection points.

2) Explain in your own words why each point on the unit circle has the coordinates (cos θ, sin θ) where θ is the degree of the angle. (Hint: draw a right triangle having the hypotenuse be the radius of one.)

3) Think about what happens as we start a second or third rotation around the unit circle. What would 390° = ? or 450° = ? Why?

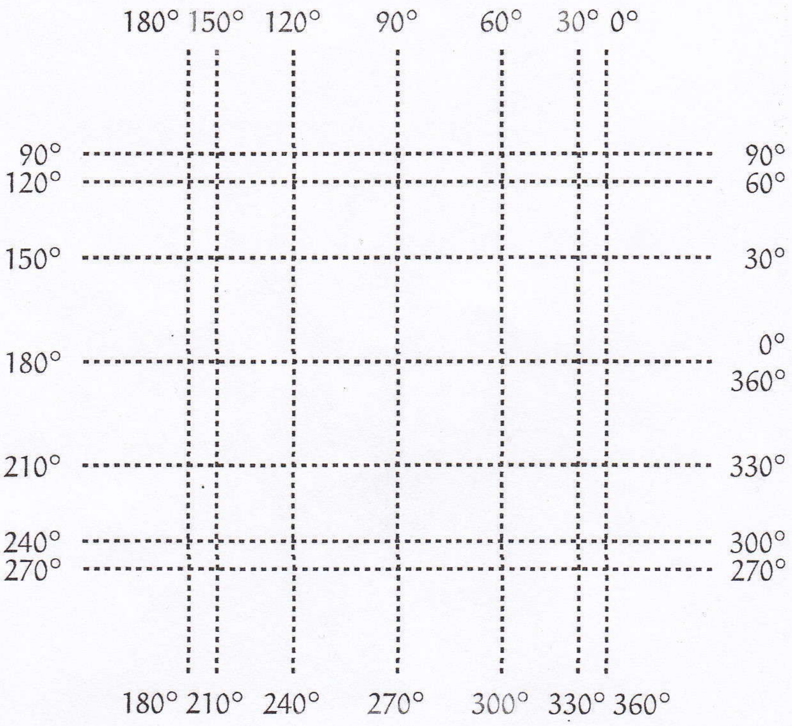
4) Taking the grid lines we created above lets draw all the points (x, y) => (cos θ, sin θ). For our work today we will only use values of θ that are multiples of 30.

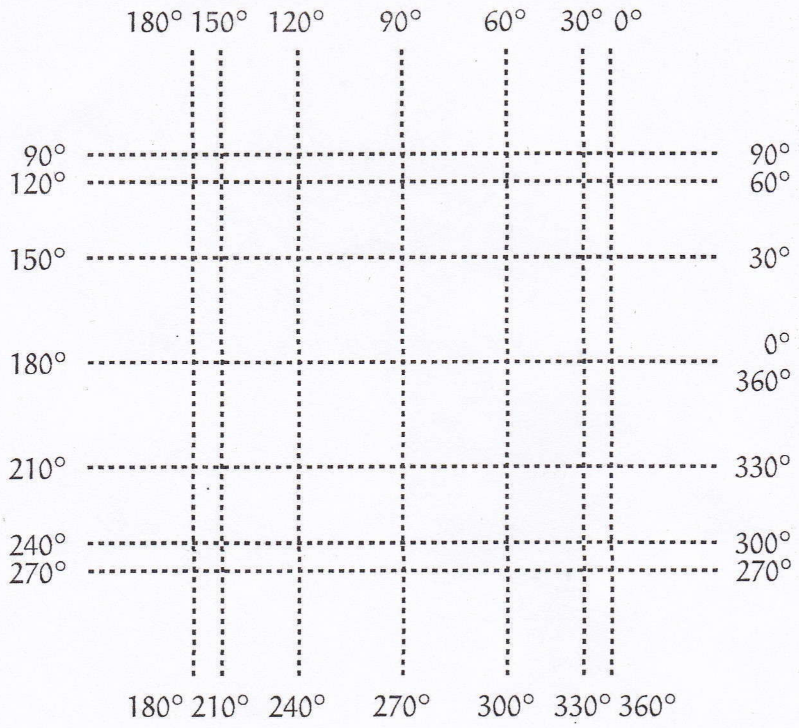
5) The following pages have many grids like the one we created. We will explore what happens as we change up what is happening to our x-coordinate (cos θ) and what happens to our y-coordinate (sin θ). Remember as we go that to place points we go to the vertical line for x and the horizontal line for y. Graph (2, 5)



Two together: These are the graphs that should be graphed. Have each person in your group do a different graph.

$(\cos\theta, \sin\theta + 30)$	$(\cos\theta, \sin\theta - 30)$	$(\cos\theta, \sin 2\theta)$	$(\cos\theta, \sin 2\theta + 90)$
$(\cos\theta, \sin\theta + 60)$	$(\cos\theta, \sin\theta - 60)$	$(\cos\theta, \sin 2\theta + 30)$	Continue adding 30°
$(\cos\theta, \sin\theta + 90)$	$(\cos\theta, \sin\theta - 90)$	$(\cos\theta, \sin 2\theta + 60)$	Try $(\cos 3\theta, \sin 2\theta)$





Teachers Notes:

These are called Lissajour Curves (LEE-suh-zhoo). Which are a parametric plot of the harmonic system. These curves have application to any repetitive, back and forth movement through an equilibrium such as sound and light waves, or electrons in a wire with alternating current. The electrical power used in TV's, radar, and microwaves are all examples. The motion of a pendulum is an example and the tuning of a motor would use these waves or curves. For example, "engineers can use the principles of Lissajour to precisely tune and set up the phase relation between a known reverence signal and a signal to be tested."

If a pendulum is made of a string connected at two points and then another string tied to that one, this will form these shapes. As one string gets longer verses the other the shapes change.
https://www.youtube.com/watch?v=uPbzhxYTioM

Above is the youtube resource that will show this and also shows how to make a pendulum for your classroom.

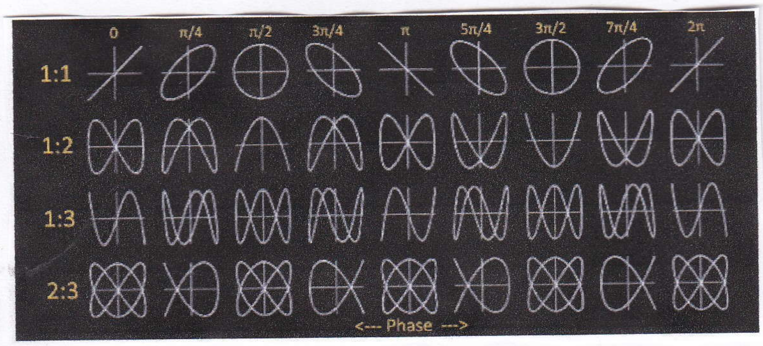
Here are the equations for Lissajour curves: $x(t) = A\sin(at + \theta)$ and $y = B\sin(bt)$

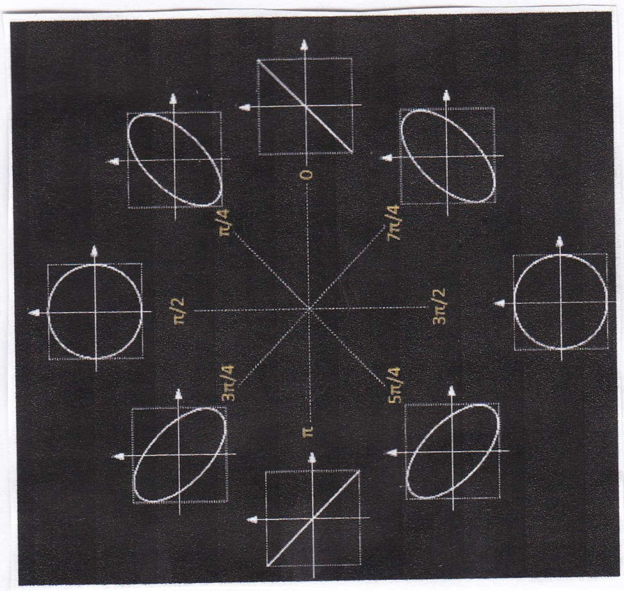
These are very similar to what we graphed, especially as we remember that

$y = \sin(x) = \cos(x - \pi/2)$. In our graphs we let A = B = 1 and did our adding to the y-coordinate instead of the x-coordinate. The behavior gets more interesting when a & b are not both equal to one. If a/b = 1 we have a line, circle, or an ellipse. The ratio a/b tells us that there are a vertical lobes and b horizontal lobes.

Talking with students about what happened as we changes our x and y-coordinates in relation to the equations given above we could talk about the amplitude, change in period of a sine and cosine function, and phase shift of the sine and cosine functions.

The following chart, on the left, gives the type of result that is formed when a/b = 1 and there is a phase shift. This chart if for the above equations and not for our (*cos x, sin x*). The chart on the right shows how the ratio gives the different number of lobes.





First two resources are interactive:

https://academo.org/demos/lissajous-curves/

http://datagenetics.com/blog/april22015/index.html

https://en.wikipedia.org/wiki/Lissajous_curve

http://www.rfcafe.com/references/popular-electronics/lissajous-had-a-figure-for-it-mar-1957-popular-electronics.htm